This book has permission to use the "N&K method of COLORS".

Question: A parallelogram is formed by the points A(4, -2), B(7, 2), C(0, 9) and D(-3, 5). If AB is the base of the parallelogram, what is the height of the parallelogram? What is the area of the parallelogram?

A) Height=$\frac{4}{5}$, $Area = 4$

B) Height=$\frac{5}{5}$, $Area = 5$

C) Height=$\frac{49}{5}$, $Area = 49$

D) Height=$\frac{94}{5}$, $Area = 94$

For speed, while solving something similar, only THINK the words in blue; WRITE only the words in other COLORS.

Given: 1) A parallelogram formed by the points A(4, -2), B(7, 2), C(0, 9) and D(-3, 5).
2) AB is the base of the parallelogram.
Solve: What is the height of the parallelogram?
What is the area of the parallelogram?

Road Map of Solution:
First Step: Create the equation for the line passing through points A(4, -2) and B(7, 2).
Second Step: Find the distance between points A(4, -2) and B(7, 2). This can be used as the BASE of the parallelogram.
Third Step: Find the distance of point C(0, 9) from the line through points A(4, -2) and B(7, 2).
This can be used as the HEIGHT of the parallelogram.

First Step: Create the equation for the line passing through points A(4, -2) and B(7, 2).
We know that the equation for the line passing through point (x_1, y_1) is given by

$$m(x-x_1) = (y-y_1) \quad \ldots\ldots\ldots\ldots \quad \text{equation \#1}$$

We also know that $m = \frac{rise}{run} = \left(\frac{(y_2-y_1)}{(x_2-x_1)}\right)$

$$\left(\frac{(y_2-y_1)}{(x_2-x_1)}\right)(x-x_1) = (y-y_1) \quad \ldots\ldots\ldots\ldots \quad \text{equation \#1b}$$

Using points A(4, -2) and B(7, 2) for
(x_1, y_1) and (x_2, y_2)
we get,

$$\left(\frac{((y_2)-(y_1))}{((x_2)-(x_1))}\right)[x-(x_1)] = [y-(y_1)]$$

$$\left(\frac{((2)-(-2))}{((7)-(4))}\right)[x-(4)] = [y-(-2)]$$

$$\left(\frac{(2\ +\ 2)}{(7\ -\ 4)}\right)[x-(4)] = [y-(-2)]$$

$$\left(\frac{(4)}{(3)}\right)[x-\ (4)] = [y-(-2)]$$

$$\frac{4(x)}{3}-\frac{4(4)}{3} = y+\ 2$$

$$\frac{4(x)}{3}-\frac{16}{3} = y+\ 2$$

$$-y\ \ -2+\left\{\frac{4(x)}{3}-\frac{16}{3}\right\} = \{y+\ 2\}\ -y\ \ -2$$

$$-y\ \ -2+\ \frac{4(x)}{3}-\frac{16}{3} = 0$$

$$\frac{4(x)}{3}-y-\frac{16}{3}-2 = 0$$

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$$\frac{4(x)}{3} - y - \frac{16}{3} - 2 \times 1 = 0$$

$$\frac{4(x)}{3} - y - \frac{16}{3} - 2 \times \frac{1}{1} = 0$$

$$\frac{4(x)}{3} - y - \frac{16}{3} - 2 \times \frac{3}{3} = 0$$

$$\frac{4(x)}{3} - y - \frac{16}{3} - \frac{2\times3}{3} = 0$$

$$\frac{4(x)}{3} - y - \frac{16}{3} - \frac{6}{3} = 0$$

$$\frac{4(x)}{3} - y - \frac{22}{3} = 0$$ *. . . . . equation #2*

equation for the line passing through points A(4, -2) and B(7, 2).

Second Step: Find the distance between points A(4, -2) and B(7, 2). This can be used as the BASE of the parallelogram.
points $A(x_1, y_1)$ and $B(x_2, y_2)$, we get,.

Distance between A and B $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[(7) - (4)]^2 + [(2) - (-2)]^2}$

$= \sqrt{[7 - 4]^2 + [2 + 2]^2}$

$= \sqrt{[3]^2 + [4]^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

Distance between A and B $= 5$ = *BASE of the parallelogram*

Third Step: Find the distance of point C(0, 9) from the line through points A(4, -2) and B(7, 2).
This can be used as the HEIGHT of the parallelogram.

We also know that
the distance "d" of point $C(x_3, y_3)$ from
the line $ax + by + c = 0$ *. . . . . equation #3*
is given by $d = \frac{|ax_3+by_3+c|}{\sqrt{a^2+b^2}}$ *. . . . . equation #4*

Using point $C(0, 9)$
for $C(x_3, y_3)$
and
line $\frac{4(x)}{3} - y - \frac{22}{3} = 0$

$$\left(\frac{4}{3}\right)x + (-1)y + \left(-\frac{22}{3}\right) = 0$$ *. . . . . equation #2b*

for $(a)x + (b)y + (c) = 0$

We get, $d = \frac{|ax_3+by_3+c|}{\sqrt{a^2+b^2}}$

$$d = \frac{\left|\left(\frac{4}{3}\right)(0)+(-1)(9)+\left(-\frac{22}{3}\right)\right|}{\sqrt{\left(\frac{4}{3}\right)^2+(-1)^2}}$$

$$d = \frac{\left|0+(-1)(9)+\left(-\frac{22}{3}\right)\right|}{\sqrt{\left(\frac{16}{9}\right)+(1)}}$$

$$d = \frac{\left|0 \quad -9 \quad -\frac{22}{3}\right|}{\sqrt{\left(\frac{16}{9}\right)+\left(\frac{9}{9}\right)}}$$

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$$d = \frac{\left| 0 - \frac{9\times3}{1\times3} - \frac{22}{3} \right|}{\sqrt{\left(\frac{16+9}{9}\right)}}$$

$$d = \frac{\left| 0 - \frac{27}{3} - \frac{22}{3} \right|}{\sqrt{\left(\frac{25}{9}\right)}}$$

$$d = \frac{\left| 0 - \frac{27+22}{3} \right|}{\sqrt{\left(\frac{25}{9}\right)}}$$

$$d = \frac{\left| 0 - \frac{49}{3} \right|}{\sqrt{\left(\frac{25}{9}\right)}}$$

$$d = \frac{\left| - \frac{49}{3} \right|}{\sqrt{\left(\frac{25}{9}\right)}}$$

$$d = \frac{\frac{49}{3}}{\sqrt{\left(\frac{25}{9}\right)}}$$

$$d = \frac{\frac{49}{3}}{\left(\frac{5}{3}\right)}$$

$$d = \left(\frac{49}{\cancel{3}}\right)\left(\frac{\cancel{3}}{5}\right)$$

$$\boxed{d = \left(\frac{49}{5}\right) \quad = \quad \textit{HEIGHT of the parallelogram}}$$

It is known that,

Area of a triangle $= \frac{1}{2} \times$ *base* $\times$ *height.*

$$= \frac{1}{2} \times \cancel{5} \times \left(\frac{49}{\cancel{5}}\right)$$

$$= \times \left(\frac{49}{2}\right)$$

Since the parallelogram is on the same Base & HEIGHT as the Triangle,

the Area of the Parallelogram = *TWICE the Area of the Triangle*

$$= \cancel{2} \times \left(\frac{49}{\cancel{2}}\right)$$

$$= 1 \left(\frac{49}{1}\right)$$

Area of the Parallelogram =	49	*Answer (C)*